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Perfect Transfer of Entangled States on Spin Chain

Jing-Ling Chen[1,2] and Qing-Liang Wang[1]

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Current researches have shown that perfect states transfer over arbitrary distances is possible for a simple unmodulated spin chain by some schemes. The transfer of a single qubit state has been investigated in detail by Christandl *et al*. [Phys. Rev. Lett. 92, 187902(2004)] through a modified Heisenberg XX model Hamiltonian H_G. The previous study of Christandl is restricted to the first-excitation states of H_G (i.e., which correspond to the second subspace of the Hilbert space of H_G). In this work, we extend their study to the case of the high-excitation states, and find that the entangled states in such a form, $|\psi\rangle = \alpha \mid 00\cdots0\rangle + \beta \mid 11\cdots1\rangle$, can be perfectly transfered on the spin chain.

KEY WORDS: perfect state transfer; entangled states; spin chain.

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1. INTRODUCTION

An important task in quantum information processing is the transfer of quantum states from one location (A) to another location (B) (Bennett and DiVincenzo, 2000). It is required, for example, to link several small quantum processors for large-scale quantum computing. Thus it is very important to have physical systems which can serve as channels for quantum communication. At the same time the problem of designing quantum networks which enable efficient high-fidelity transfer of quantum states has also recently been addressed by a number of authors (Bose, 2003; Christandl *et al*., 2004; Albanese *et al*., 2004; Yung and Bose, 2005; Shi *et al*., 2005; De Chiara *et al*., 2005), so there have been many proposals of physical systems to serve as channels for quantum communication (Kielpinski *et al*., 2002), but, the spin chains(quantum network) (Bose, 2003; DiVincenzo *et al*., 2000) is the most typical one. A 1D quantum network was proposed by Bose (2003), who considered a spin chain with the nearest-neighbor Heisenberg Hamiltonian; here, the transmission of quantum state between the ends of the

[1] Theoretical Physics Division, Chern Institute of Mathematics, Center for Combinatorics and LPMC, Nankai University, Tianjin 300071, P.R. China.
[2] To whom correspondence should be addressed; e-mail: chenjl@nankai.edu.en

chain was achieved simply by a free evolution of the network. Similarly, in Shi *et al*. (2005) and Christandl *et al*. (2005), nearly perfect state transfer was achieved for uniform couplings provided a spatially varying magnetic field was introduced. Wójcik *et al*. (2005) studied a transfer of quantum states between two qubits attached to the ends of a quantum wire consisting of N linearly arranged spins. In Karbach and Stolze (2005) and Albanese *et al*., 2004 mirror inversion of quantum states'is introduced. For the first-excitation subspace perfect transfer of arbitrary states on quantum spin networks has been investigated by Christandl *et al*. (2004, 2005).

The previous study of Christandl *et al*. is restricted to the first-excitation states of H_G, where H_G is the Hamiltonian of a modified Heisenberg XX model (the first-excitation states correspond to the second subspace of the Hilbert space of H_G). The purpose of our work is to extend their study to the case of the high-excitation states, and we find that the entangled states in such a form, $|\psi\rangle = \alpha\,|\,00\cdots0\rangle + \beta\,|\,11\cdots1\rangle$, can be perfectly transfered on the spin chain. The paper is organized as follows. In Section 2, we present the Hamiltonian H_G of the modified Heisenberg XX model. In Section 3, we make a brief review for the perfect transfer of a single qubit state through the Hamiltonian H_G. In Section 4, we extend the perfect state transfer problem to an entangled state of two-qubit in the following form, $|\psi\rangle = \alpha\,|\,00\rangle + \beta\,|\,11\rangle$. Some conclusion and discussion are made in the last section.

2. THE HAMILTONIAN

Let us consider the Hamiltonian of a modified Heisenberg XX model:

$$H_G = \sum_{n=1}^{N-1} J_{n,n+1}\left(\sigma_n^x\sigma_{n+1}^x + \sigma_n^y\sigma_{n+1}^y\right) = \frac{1}{2}\sum_{n=1}^{N-1} J_{n,n+1}\left(\sigma_n^+\sigma_{n+1}^- + \sigma_n^-\sigma_{n+1}^+\right), \quad (1)$$

where $J_{n,n+1} = \sqrt{n(N-n)}$ are coupling strengths between lattices n and $n+1$, σ_n^i and σ_{n+1}^i are the Pauli matrices acting on the n-th and $(n+1)$-th qubit with $i = x, y, z$. Obviously, the Hamiltonian H_G describes a nearest-neighbor interaction spin chain. Hamiltonian H_G has 2^N complete and orthogonal eigenvectors, which span the Hilbert space of H_G. The Hilbert space of H_G can be divided into $N+1$ subspaces based on the population of reversed spin. The first subspace has only one (because $C_N^0 = 1$) eigenvector with zero-value of eigenvalue, i.e.

$$|\psi_0\rangle = |00\cdots0\rangle, \quad H_G|\psi_0\rangle = E_0|\psi_0\rangle, \quad E_0 = 0, \quad (2)$$

where we have denoted $|0\rangle$ as the state of spin-down $|\downarrow\rangle$, and $|1\rangle$ as the state of spin-up $|\uparrow\rangle$. The (ground) state $|\psi_0\rangle$ is a state with all spins down.

The second subspace contains N (because $C_N^1 = N$) first-excitation states, which have the following forms

$$|\psi_1\rangle^{(k)} = \sum_{m=1}^{N} a_k(m)\phi(m), \quad H_G|\psi_1\rangle^{(k)} = E_1^{(k)}|\psi_1\rangle^{(k)}, \quad k = 1, 2, \ldots, N, \quad (3)$$

where

$$\phi(m) = |00\cdots 1_m \cdots 0\rangle \quad (4)$$

represents a state in which only the spin on the m-th lattice is up.

The third subspace contains $N(N-1)/2$ (because $C_N^2 = N(N-1)/2$) second-excitation states, which have the following forms

$$|\psi_2\rangle^{(k)} = \sum_{m1<m2}^{N} a_k(m1, m2)\phi(m1, m2), \quad H_G|\psi_2\rangle^{(k)} = E_2^{(k)}|\psi_2\rangle^{(k)},$$
$$k = 1, 2, \ldots, N(N-1)/2, \quad (5)$$

where

$$\phi(m1, m2) = |\cdots 1_{m1} \cdots 1_{m2} \cdots\rangle \quad (6)$$

represents a state in which only the spins on the $m1$-th and $m2$-th lattices are up. Similarly the j-th subspace contains $C_N^j = \frac{N!}{j!(N-j)!}$ states, which we will not list here.

Due to the complete and orthonormal basis states of H_G, we can write H_G as

$$\begin{aligned} H_G = &\sum_{k_0=1}^{1} E_0|\psi_0><\psi_0| + \sum_{k_1=1}^{N} E_1^{(k_1)}|\psi_1>^{(k_1)\ (k_1)}<\psi_1| \\ &+ \sum_{k_2=1}^{N(N-1)/2} E_2^{(k_2)}|\psi_2>^{(k_2)\ (k_2)}<\psi_2| \\ &+ \cdots + \sum_{k_N=1}^{1} E_N^{(k_N)}|\psi_N>^{(k_N)\ (k_N)}<\psi_N|, \end{aligned} \quad (7)$$

from which we obtain the unitary evolution operator of H_G as

$$\begin{aligned} e^{-i\lambda t H_G} = &\sum_{k_0=1}^{1} e^{-i\lambda t E_0}|\psi_0><\psi_0| + \sum_{k_1=1}^{N} e^{-i\lambda t E_1^{(k_1)}}|\psi_1>^{(k_1)\ (k_1)}<\psi_1| \\ &+ \sum_{k_2=1}^{N(N-1)/2} e^{-i\lambda t E_2^{(k_2)}}|\psi_2>^{(k_2)\ (k_1)}<\psi_2| \\ &+ \cdots + \sum_{k_N=1}^{1} e^{-i\lambda t E_N^{(k_N)}}|\psi_N>^{(k_N)\ (k_N)}<\psi_N|. \end{aligned} \quad (8)$$

3. TRANSFER OF A SINGLE QUBIT STATE

In the case for the first-excitation (Christandl *et al*., 2004, 2005) has investigated perfect transfer of arbitrary states in the second subspace on the quantum spin networks. Suppose we prepare the input qubit A in state $\alpha|0\rangle + \beta|1\rangle$, the state of the network becomes

$$\alpha|0_A 00\cdots 00_B\rangle| + \beta|1_A 00\cdots 00_B\rangle = \alpha|\underline{0}\rangle + \beta|1\rangle. \tag{9}$$

The coefficient α does not change in time as $|\underline{0}\rangle$ is the zero energy eigenstates of H_G. Since the operator of the total z-component of the spin, $\sigma_{tot}^z = \sum_{j=1}^N \sigma_j^z$, commutes with H_G which leads to the conservation of the total z-component of spin. Therefore the state $|1\rangle = |1_A 00\cdots 00_B\rangle$ will evolve into a superposition of states with exactly one spin "up" and all other spins "down". Thus the initial state of the network evolves in time t as

$$\alpha|\underline{0}\rangle + \beta|1\rangle \to \alpha|\underline{0}\rangle + \sum_{n=1}^{N} \beta_n(t)|n\rangle. \tag{10}$$

The dynamics is effectively confined to second subspace S_G. If we identify qubit A with vertex 1 and qubit B with vertex N then all we want to know is the probability amplitude that the network initially in state $|1\rangle$, corresponding to $|1_A 00\cdots 00_B\rangle$, evolves after time t to state $|N\rangle$, corresponding to $|0_A 00\cdots 01_B\rangle$), i.e.

$$F(t) = \langle N|e^{-i\lambda H_G}|1\rangle. \tag{11}$$

More precisely, with Eq. (8), we have (obviously, only the eigenvectors in the second subspace have contributions)

$$\begin{aligned} F(t) &= \langle N|e^{-i\lambda t H_G}|1\rangle \\ &= \sum_{k=1}^{N} a_k^*(1)a_k(N)e^{-i\lambda t E_1^{(k)}}. \end{aligned} \tag{12}$$

The faithful state transfer is obtained for some certain times t such that $|F(t)| = 1$. Christandl *et al*. (2004) has the following result:

Theorem 3.1. *For the quantum walk concerning the first-excitation state,*

$$F(t) = \left[-i \sin\left(\frac{\lambda t}{2}\right)\right]^{N-1} \tag{13}$$

Proof: Here we would like to provide an analytic proof through the language of matrix. As shown in Eq. (3), the first-excitation states have the following form

$$|\psi_1\rangle^{(k)} = \sum_{m=1}^{N} a_k(m)|00\cdots 1_m \cdots 0\rangle, \quad k = 1, 2, \ldots, N. \tag{14}$$

We can associate N matrix columns correspond to N states $|100\cdots 0\rangle$, $|010\cdots 0\rangle, \cdots, |0\cdots 01\rangle$ respectively. Namely

$$\begin{pmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{pmatrix} \longrightarrow |100\cdots 0\rangle, \quad \begin{pmatrix} 0 \\ 1 \\ \vdots \\ 0 \end{pmatrix} \longrightarrow |01\cdots 0\rangle, \cdots, \quad \begin{pmatrix} 0 \\ 0 \\ \vdots \\ 1 \end{pmatrix} \longrightarrow |00\cdots 01\rangle. \tag{15}$$

When considering the first excitation states, we can restrict the Hamiltonian H_G to the second subspace. At this moment, from the point of view of the matrix language, the Hamiltonian H_G corresponds to the following $N \times N (C_N^1 = N)$ matrix

$$H_G = \frac{1}{2}\begin{pmatrix} 0 & J_{12} & 0 & 0 & \cdots & 0 \\ J_{12} & 0 & J_{23} & 0 & \cdots & 0 \\ 0 & J_{23} & 0 & J_{34} & \cdots & 0 \\ 0 & 0 & J_{34} & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & J_{N-1,N} \\ 0 & 0 & 0 & 0 & J_{N-1,N} & 0 \end{pmatrix} \tag{16}$$

and the first excitation states becomes

$$|\psi_1\rangle^{(k)} = \begin{pmatrix} a_k(1) \\ a_k(2) \\ \vdots \\ a_k(N) \end{pmatrix}, \tag{17}$$

with corresponding eigenvalues $E_1^{(k)}$. It means that the matrix H_G can be diagonalized in the following way

$$H_G = U\ H^{\text{diag}} U^{\dagger}, \tag{18}$$

where

$$H^{\text{diag}} = \begin{pmatrix} E_1^{(1)} & 0 & 0 & 0 & \cdots & 0 \\ 0 & E_1^{(2)} & 0 & 0 & \cdots & 0 \\ 0 & 0 & E_1^{(3)} & 0 & \cdots & 0 \\ 0 & 0 & 0 & E_1^{(4)} & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & 0 & 0 & E_1^{(N)} \end{pmatrix}, \tag{19}$$

$$U = \begin{pmatrix} a_1(1) & a_2(1) & a_3(1) & a_4(1) & \cdots & a_N(1) \\ a_1(2) & a_2(2) & a_3(2) & a_4(2) & \cdots & a_N(2) \\ a_1(3) & a_2(3) & a_3(3) & a_4(3) & \cdots & a_N(3) \\ a_1(4) & a_2(4) & a_3(4) & a_4(4) & \cdots & a_N(4) \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ a_1(N) & a_2(N) & a_3(N) & a_4(N) & \cdots & a_N(N) \end{pmatrix} \tag{20}$$

(Obviously, $|\psi_1\rangle^{(k)}$ forms the k-th column of the unitary matrix U). With Eqs. (19) and (20), it is easy to show

$$\begin{aligned} F(t) &= (e^{-i\lambda t H_G})_{N,1} = \langle N|e^{-i\lambda t H_G}|1\rangle \\ &= \langle 0\cdots 01|Ue^{-i\lambda H^{\text{diag}}}U^{\dagger}|10\cdots 0\rangle = \sum_{k=1}^{N} a_k^*(1)a_k(N)e^{-i\lambda t E_1^{(K)}}, \end{aligned} \tag{21}$$

where $(e^{-i\lambda H_G})_{N,1}$ represents a matrix element in the N-th row and 1-st column of the matrix $e^{-i\lambda t H_G}$. It is obvious that $F(t=0) = \sum_{k=1}^{N} a_k^*(1)a_k(N) = 0$, this comes from the orthogonal property of the unitary matrix U. □

In principle, we can determine the function $F(t)$ if the eigenvectors $|\psi_1\rangle^{(k)}$ and eigenvalues $E_1^{(k)}$ were known. However, the Hamiltonian H_G becomes the x-component of angular momentum operator S_x when the coupling strength $J_{n,n+1} = \sqrt{n(N-n)}$. This fact enables us to determine $F(t)$ without solving the eigenproblem $H|\psi\rangle = E|\psi\rangle$. The eigenvalues of S_x are the same with S_z, i.e. $E \in \{\frac{N-1}{2}, \frac{N-3}{2}, \cdots, -\frac{N-1}{2}\}$. For angular momentum, there is a disentangle formula:

$$\exp(re^{i\varphi}S_+ - re^{-i\varphi}S_-) = \exp(e^{i\varphi}\tan r S_+)(1+\tan^2 r)^{S_z}\exp(-e^{i\varphi}\tan r S_-). \tag{22}$$

Since $H_G = S_x = (S_+ + S_-)/2$, for $r = \frac{\lambda t}{2}$, $\varphi = -\frac{\pi}{2}$ we then have

$$\begin{aligned} e^{-i\lambda t H_G} &= e^{-i\lambda t S_x} = \exp(re^{-i\pi/2}S_+ - re^{i\pi/2}S_-) \\ &= \exp(-i\tan r S_+)(1+\tan^2 r)^{S_z}\exp(-i\tan r S_-). \end{aligned} \tag{23}$$

It is easy to know that ($S_+^N = 0$)

$$\exp(-i\tan r S_+) = 1 + \frac{-i\tan r}{1!}S_+ + \frac{(-i\tan r)^2}{2!}S_+^2 + \cdots + \frac{(-i\tan r)^{N-1}}{(N-1)!}S_+^{N-1} \tag{24}$$

is a upper-triangle matrix. The matrix elements are

$$[\exp(-i\tan r S_+)]_{N,N} = 1, \tag{25}$$

$$[\exp(-i\tan r S_+)]_{1,N} = (J_{12}J_{23}J_{34}\cdots J_{N,N-1})\frac{(-i\tan r)^{n-1}}{(N-1)!} = (-i\tan r)^{N-1},$$

$$[\exp(-i\tan r S_-)]_{N,1} = (-i\tan r)^{N-1}. \tag{26}$$

$(1+\tan^2 r)^{S_z}$ is a diagonal matrix, where the matrix element

$$[(1+\tan^2 r)^{S_z}]_{N,N} = (\cos^{-2} r)^{-(N-1)/2} = [\cos r]^{N-1}. \tag{27}$$

Consequently, one has

$$\begin{aligned} F(t) &= \langle 00\cdots 01|e^{-i\lambda t H_G}|10\cdots 00\rangle = (e^{-i\lambda t H_G})_{N,1} \\ &= [\exp(-i\tan r S_+)]_{N,N}[(1+\tan^2 r)^{S_z}]_{N,N}[\exp(-i\tan r S_-)_{N,1} \\ &= 1\cdot[\cos r]^{N-1}\cdot(-i\tan r)^{N-1} = [-i\sin r]^{N-1} \\ &= \left[-i\sin\left(\frac{\lambda t}{2}\right)\right]^{N-1}. \end{aligned} \tag{28}$$

This ends the proof. Obviously, for times $t = \pi/\lambda$, one has the perfect state transfer $F(t) = 1$.

4. TRANSFER OF AN ENTANGLED STATE OF TWO-QUBIT

In this section, we extend the perfect state transfer problem to an entangled state of two-qubit in the following form, $|\psi\rangle = \alpha|00\rangle + \beta|11\rangle$. Suppose we prepare the input qubits A1 and A2 in state $\alpha|0_{A_1}0_{A_2}\rangle + \beta|1_{A_1}1_{A_2}\rangle$ the state of the network becomes

$$\alpha|0_{A_1}0_{A_2}0\cdots 00_{B_1}0_{B_2}\rangle + \beta|1_{A_1}1_{A_2}0\cdots 00_{B_1}0_{B_2}\rangle = \alpha|\underline{00}\rangle + \beta|11\rangle. \tag{29}$$

And then we have the $F(t)$ as

$$F(t) = \langle M|e^{-i\lambda t H_G}|1\rangle. \tag{30}$$

More precisely, with Eq. (8), we have (where $M = C_N^2 = N(N-1)/2$)

$$F(t) = \langle M|e^{-i\lambda t H_G}|1\rangle = \sum_{k=1}^{M} b_k^*(1)b_k(M)e^{-i\lambda t E_2^{(k)}}. \tag{31}$$

Our main result is

Main Result: For the quantum walk concerning the second-excitation states,

$$F(t) = \left[-i \sin\left(\frac{\lambda t}{2}\right)\right]^{2(N-2)} \tag{32}$$

In the following we would like to determine the function $F(t)$ by exactly solve the first few cases with $N = 2, 3, 4, 5$ with Eq. (31).

$\mathbf{N} = 2$: In this case, obviously $F(t) = 1$.

$\mathbf{N} = 3$: In this case

$$H_G = \begin{pmatrix} 0 & J_{23} & 0 \\ J_{23} & 0 & J_{12} \\ 0 & J_{12} & 0 \end{pmatrix} = \frac{1}{2}\begin{pmatrix} 0 & \sqrt{2} & 0 \\ \sqrt{2} & 0 & \sqrt{2} \\ 0 & \sqrt{2} & 0 \end{pmatrix}, \tag{33}$$

$$H^{\text{diag}} = \begin{pmatrix} E_2^{(1)} & 0 & 0 \\ 0 & E_2^{(2)} & 0 \\ 0 & 0 & E_2^{(3)} \end{pmatrix} = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \tag{34}$$

$$U = \begin{pmatrix} b_1(1) & b_2(1) & b_3(1) \\ b_1(2) & b_2(2) & b_3(2) \\ b_1(3) & b_2(3) & b_3(3) \end{pmatrix} = \begin{pmatrix} \frac{1}{2} & -\frac{1}{\sqrt{2}} & \frac{1}{2} \\ -\frac{\sqrt{2}}{2} & 0 & \frac{\sqrt{2}}{2} \\ \frac{1}{2} & \frac{1}{\sqrt{2}} & \frac{1}{2} \end{pmatrix} \tag{35}$$

(where $b_k(1) = a_k(1, 2)$, $b_k(2) = a_k(1, 3)$, $b_k(3) = a_k(2, 3)$), from which one obtains

$$F(t) = (e^{-i\lambda t H_G})_{M,1} = \sum_{k=1}^{3} b_k^*(1) b_k(M) e^{-i\lambda E_2^{(k)}} = \left[-i \sin\left(\frac{\lambda t}{2}\right)\right]^2. \tag{36}$$

$\mathbf{N} = 4$: In this case

$$H_G = \begin{pmatrix} 0 & J_{23} & 0 & 0 & 0 & 0 \\ J_{23} & 0 & J_{34} & J_{12} & 0 & 0 \\ 0 & J_{34} & 0 & 0 & J_{12} & 0 \\ 0 & J_{12} & 0 & 0 & J_{34} & 0 \\ 0 & 0 & J_{12} & J_{34} & 0 & J_{23} \\ 0 & 0 & 0 & 0 & J_{23} & 0 \end{pmatrix} = \begin{pmatrix} 0 & 2 & 0 & 0 & 0 & 0 \\ 2 & 0 & \sqrt{3} & \sqrt{3} & 0 & 0 \\ 0 & \sqrt{3} & 0 & 0 & \sqrt{3} & 0 \\ 0 & \sqrt{3} & 0 & 0 & \sqrt{3} & 0 \\ 0 & 0 & \sqrt{3} & \sqrt{3} & 0 & 2 \\ 0 & 0 & 0 & 0 & 2 & 0 \end{pmatrix}. \tag{37}$$

$$H^{\text{diag}} = \begin{pmatrix} E_2^{(1)} & 0 & 0 & 0 & 0 & 0 \\ 0 & E_2^{(2)} & 0 & 0 & 0 & 0 \\ 0 & 0 & E_2^{(3)} & 0 & 0 & 0 \\ 0 & 0 & 0 & E_2^{(4)} & 0 & 0 \\ 0 & 0 & 0 & 0 & E_2^{(5)} & 0 \\ 0 & 0 & 0 & 0 & 0 & E_2^{(5)} \end{pmatrix} = \begin{pmatrix} -2 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2 \end{pmatrix}, \tag{38}$$

$$U = \begin{pmatrix} 1/4 & -1/2 & \sqrt{3/10} & -\sqrt{3}/\sqrt{40} & -1/2 & 1/4 \\ -2/4 & 1/2 & 0 & 0 & -1/2 & 2/4 \\ \sqrt{3}/4 & 0 & -2\sqrt{10} & -3/\sqrt{40} & 0 & \sqrt{3}/4 \\ \sqrt{3}/4 & 0 & 0 & 5/\sqrt{40} & 0 & \sqrt{3}/4 \\ -2/4 & -1/2 & 0 & 0 & 1/2 & 2/4 \\ 1/4 & 1/2 & \sqrt{3/10} & -\sqrt{3}/\sqrt{40} & 1/2 & 1/4 \end{pmatrix}, \tag{39}$$

$$F(t) = (e^{i\lambda t H_G})_{M,1} = \sum_{k=1}^{6} b_k^*(1) b_k(6) e^{-i\lambda E_2^{(k)}} = \left[-i \sin\left(\frac{\lambda t}{2}\right) \right]^4. \tag{40}$$

$\mathbf{N} = 5$: In this case

$$H_G = \begin{pmatrix} 0 & J_{23} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ J_{23} & 0 & J_{34} & 0 & J_{12} & 0 & 0 & 0 & 0 & 0 \\ 0 & J_{34} & 0 & J_{45} & 0 & J_{12} & 0 & 0 & 0 & 0 \\ 0 & 0 & J_{45} & 0 & 0 & 0 & J_{12} & 0 & 0 & 0 \\ 0 & J_{12} & 0 & 0 & 0 & J_{34} & 0 & 0 & 0 & 0 \\ 0 & 0 & J_{12} & 0 & J_{34} & 0 & J_{45} & J_{23} & 0 & 0 \\ 0 & 0 & 0 & J_{12} & 0 & J_{45} & 0 & 0 & J_{23} & 0 \\ 0 & 0 & 0 & 0 & 0 & J_{23} & 0 & 0 & J_{45} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & J_{23} & J_{45} & 0 & J_{34} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & J_{34} & 0 \end{pmatrix}$$

$$= \frac{1}{2}\begin{pmatrix} 0 & \sqrt{6} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \sqrt{6} & 0 & \sqrt{6} & 0 & 2 & 0 & 0 & 0 & 0 & 0 \\ 0 & \sqrt{6} & 0 & 2 & 0 & 2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 & 0 & 2 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 & 0 & \sqrt{6} & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & \sqrt{6} & 0 & 2 & \sqrt{6} & 0 & 0 \\ 0 & 0 & 0 & 2 & 0 & 2 & 0 & 0 & \sqrt{6} & 0 \\ 0 & 0 & 0 & 0 & 0 & \sqrt{6} & 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \sqrt{6} & 2 & 0 & \sqrt{6} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \sqrt{6} & 0 \end{pmatrix}, \quad (41)$$

$$H^{\text{diag}} = \begin{pmatrix} -3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3 \end{pmatrix}, \quad (42)$$

$$U = \begin{pmatrix} 1/8 & -\sqrt{3/2}/4 & 3/(2\sqrt{10}) & -\sqrt{3/5}/8 & -1/2 & 1/4 & 3/(2\sqrt{10}) & -\sqrt{3/5}/8 & -\sqrt{3/2}/4 & 1/8 \\ -\sqrt{3/2}/4 & 1/2 & -\sqrt{3/5/2} & 1/(4\sqrt{10}) & 0 & 0 & \sqrt{3/5}/2 & -1/(4\sqrt{10}) & -1/2 & \sqrt{3/2}/4 \\ 3/8 & -\sqrt{3/2}/4 & -1/(2\sqrt{10}) & -(3\sqrt{3/5})/8 & 1/2 & 1/4 & -1(2\sqrt{10}) & -(3\sqrt{3/5})/8 & -\sqrt{3/2}/4 & 3/8 \\ -1/4 & 0 & 1/\sqrt{10} & (3\sqrt{3/5})/4 & 0 & 0 & -1/\sqrt{10} & -(3\sqrt{3/5})/4 & 0 & 1/4 \\ \sqrt{3/2}/4 & -1/4 & 0 & \sqrt{5/2}/4 & 0 & -\sqrt{3/2}/2 & 0 & \sqrt{5/2}/4 & -1/4 & \sqrt{3/2}/4 \\ -1/2 & 0 & 1/\sqrt{10} & -\sqrt{3/5}/2 & 0 & 0 & -1/\sqrt{10} & \sqrt{3/5}/2 & 0 & 1/2 \\ 3/8 & \sqrt{3/2}/4 & -1/(2\sqrt{10}) & -(3\sqrt{3/5})/8 & -1/2 & -1/4 & -1/(2\sqrt{10}) & -(3\sqrt{3/5})/8 & \sqrt{3/2}/4 & 3/8 \\ \sqrt{3/2}/4 & 1/4 & 0 & \sqrt{5/2}/4 & 0 & \sqrt{3/2}/2 & 0 & \sqrt{5/2}/4 & 1/4 & \sqrt{3/2}/4 \\ -\sqrt{3/2}/4 & -1/2 & -\sqrt{3/5}/2 & 1/(4\sqrt{10}) & 0 & 0 & \sqrt{3/5}/2 & -1/(4\sqrt{10}) & 1/2 & \sqrt{3/2}/4 \\ 1/8 & \sqrt{3/2}/4 & 3/(2\sqrt{10}) & -\sqrt{3/5}/8 & 1/2 & -1/4 & 3/(2\sqrt{10}) & -\sqrt{3/5}/8 & \sqrt{3/2}/4 & 1/8 \end{pmatrix}$$

(43)

$$F(t) = (e^{-i\lambda t H_G})_{M,1} = \sum_{k=1}^{10} b_k^*(1) b_k(10) e^{-i\lambda E_2^{(k)}} = \left[-i \sin\left(\frac{\lambda t}{2}\right)\right]^6. \qquad (44)$$

For times $t = \pi/\lambda$, one has the perfect state transfer $F(t) = 1$.

5. DISCUSSION AND CONCLUSION

Similarly, for the quantum walk concerning m-th excitation states, one will have the result $F(t) = [-i \sin(\lambda t/2)]^{m(N-m)}$ for the transfer of the entangled state in such a form $|\psi\rangle = \alpha|00\cdots0\rangle + \beta|11\cdots1\rangle$. The calculation becomes more leanthy, which we would like to omit here. The faithful state transfers $F(t) = 1$ are all obtained for the times $t = \pi/\lambda$. In conclusion, we have extended the previous study of Christandl to the case of the high-excitation states, and find that the entangled states in such a form, $|\psi\rangle = \alpha|00\cdots0\rangle + \beta|11\cdots1\rangle$, can be perfectly transfered on the spin chain.

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